Journal of Engineering Physics and Thermophysics, Vol. 78, No. 5, 2005

INFLUENCE OF INERTIAL EFFECTS ON THE DISINTEGRATION OF THE STRUCTURE OF DISPERSE SYSTEMS OF THE SOLID-PHASE–LIQUID-MEDIUM TYPE

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UDC 541.182

The conditions of disintegration of structured formations occurring in disperse systems of the solid-dispersed-phase–liquid-dispersion-medium type, increasing their effective viscosity and inhibiting the attainment of a high fluidity by the dispersions, have been investigated. The role of inertial effects in attainment of the isotropic dynamic state by S/L disperse systems, when external vibration action is applied to them, has been studied. It has been shown in what manner the presence of particles of different mass (and consequently inertia) in the system leads to a disintegration of the structure under the action of dispersion-medium oscillations.

Introduction. The attainment of a high fluidity by disperse systems of the solid-phase–liquid-medium (S/L) type is ensured due to the realization of an isotropic dynamic state eliminating the occurrence of inhomogeneities in the form of aggregates and structured layers in motion of the dispersion. The disintegration of such structural elements in disperse systems with particles larger than 1 μm in size, which are incapable of disintegrating under the action of Brownian motion of particles, may be carried out by imparting forced vibrations from an external source to the system [1]. The problem on determination of the parameters of vibration actions necessary for disintegration of a coagulation structure is posed in [2–4], where the necessary condition of attainment of the isotropism of structural disintegration is also given. What this condition means is that the energy supplied to a disperse system and transferred by the particles fixed in the structural network must be higher than the total energy of bond of the particles with the neighboring particles.

In this work, consideration is given to another aspect of the problem of disintegration of structured formations inhibiting the high fluidity of the dispersion, namely, the role of inertial effects in attainment of the isotropic dynamic state by S/L disperse systems.

Equation of Motion of a Free Particle in an Oscillating Liquid. A number of aspects of the motion of a free particle in an oscillating liquid were the focus of [5, 6]. Solution of this problem in general form requires that many effects be allowed for. Thus, the influence of the inertial component of the friction force acting on a fast moving particle on the source side of an ideal liquid can be expressed by the increase in the effective mass of the particle by the value of the additional mass of the liquid [7, 8]. The slippage of a medium relative to the particle surface is allowed for by introduction of the corresponding coefficient. The slip effect is the most pronounced for particles with a lyophobic surface that move with higher velocities relative to the dispersion medium [9]. In the present work, consideration is given to the simplest case where only the Stokes viscous force acts on a particle and there is no slippage between the medium and the particle. A certain simplification of the actual situation does not largely change the pattern of the process but enables one to obtain the analytical solutions of the equations of motion, thus demonstrating most easily and clearly the main idea of the work.

We will assume that the harmonic oscillations of a liquid medium in the direction of the vertical coordinate y follow the law

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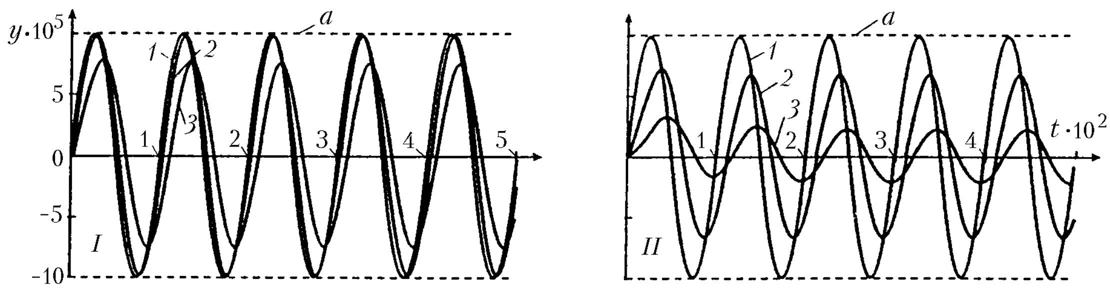


Fig. 1. Oscillations of particles of different size under the action of oscillation of the dispersion medium of different viscosity (ρ = 2600 kg/m^3, ν = 100 Hz, and a = $1\cdot10^{-4}$ m, 1) d = 10; 2) 50; 3) 100 μm): I) η = $1\cdot10^{-3}$ Pa·sec; II) η = $2\cdot10^{-4}$ Pa·sec.

$$y^{\text{liq}} = a \sin(\omega t) . \tag{1}$$

Then the speed of such oscillations is equal to

$$v^{\text{liq}} = a\omega \cos(\omega t) . \tag{2}$$

Let us consider a single particle of mass m contained in a liquid which executes harmonic oscillations by the law (1). A viscous force which is in proportion to the difference of the liquid and particle velocities and makes the particle execute oscillatory motions will act on the particle on the source side of the liquid in the laminar-regime approximation. We write the equation of motion of the particle:

$$m \frac{dv}{dt} = -3\pi\eta d (v - v^{\text{liq}}) . \tag{3}$$

If it is necessary to allow for the inertial component of the force of resistance from the liquid, instead of the quantity m in Eq. (3) we should use the sum of the particle mass and the additional liquid mass, which is equal to half the mass of the displaced medium in the case of a spherical particle: $m = m_{\text{p}} + \pi d^3 \rho_{\text{liq}}/12$.

We will assume that the particle is stationary at the initial instant of time, i.e., $v(0) = 0$. Substituting expression (2) into Eq. (3) and introducing the notation $k_1 = -3\pi\eta d/m$ and $k_2 = -a\omega k_1$, we obtain the following differential equation:

$$\frac{dv}{dt} = k_1 v + k_2 \cos(\omega t) . \tag{4}$$

The solution of Eq. (4) is the expression for the particle velocity:

$$v(t) = \frac{k_1 k_2 \exp(k_1 t) - k_2 [k_1 \cos(\omega t) - \omega \sin(\omega t)]}{k_1^2 + \omega^2} . \tag{5}$$

Integration of Eq. (5) yields the dependence of the particle's coordinate on time:

$$y(t) = \frac{k_2 [\omega \exp(k_1 t) - k_1 \sin(\omega t) - \omega \cos(\omega t)]}{\omega (k_1^2 + \omega^2)} . \tag{6}$$

Using Eqs. (5) and (6), we can evaluate the influence of a number of parameters on the capacity of the particle for executing oscillatory motions under the action of dispersion-medium oscillations.

Figure 1 shows the oscillations (calculated from Eq. (6)) of particles of different diameters in the dispersion medium with a varying viscosity. The calculations show that, as the diameter increases, the increase in the mass and

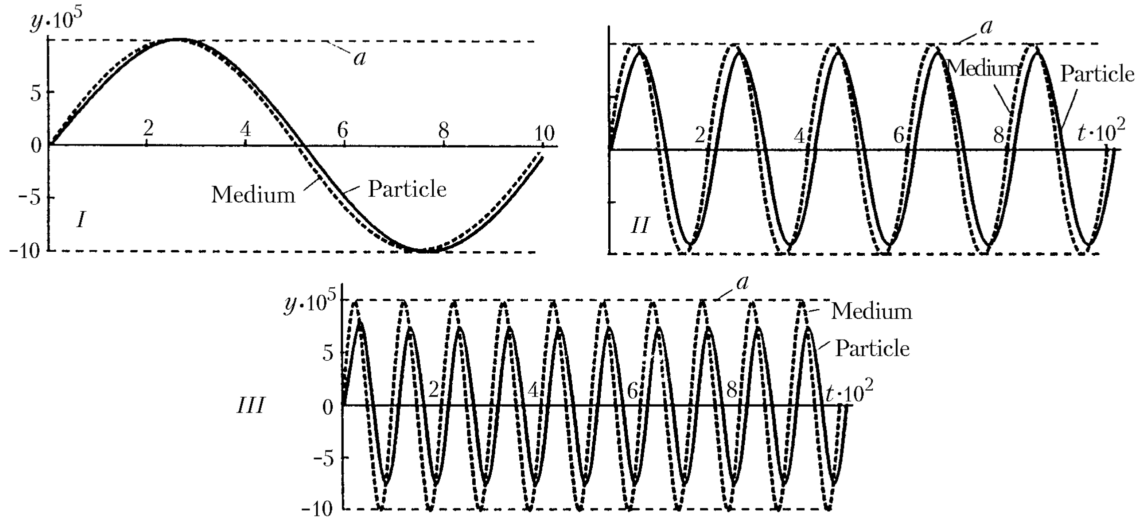


Fig. 2. Influence of the vibration frequency of the dispersion medium on the oscillation amplitude of particles (d = 100 μm, $\eta = 1\cdot10^{-3}$ Pa·sec, $a = 1\cdot10^{-4}$ m, and $\rho = 2600$ kg/m^3): I) ν = 10; II) 50; III) 100 Hz.

consequently inertia of the particles leads to a decrease in their oscillation amplitude. Thus, particles of diameters 10 and 50 μm (Fig. 1 I, curves 1 and 2) move virtually in step with the oscillations of the medium. The amplitude of their motion almost totally coincides with the amplitude of liquid oscillations. A particle of size 100 μm, due to its large mass and inertia, lags behind the oscillations of the medium. It is clear from the figure that not only are the oscillations of this particle executed with a smaller amplitude but they are somewhat out of phase. This suggests the time lag of the oscillations of the particle in relation to the oscillating liquid. Such a lag is also attributable to the increase in the relaxation time of particles with diameter; this increase becomes comparable to the time of action on a particle ($\tau_i \sim \rho d^2/\eta = 2\cdot10^3\cdot(10^{-4})^2/10^{-3} = 2\cdot10^{-2}$; the period of oscillations of the liquid is $\tau_{liq} = 1/\nu = 10^{-2}$). The reduction in the medium's viscosity (Fig. 1 II) leads to a decrease in the viscous force acting on the source side of the moving liquid on a particle, due to which the inertial effects in the system become more pronounced. The lag of the oscillations of particles behind those of the medium manifests itself for a smaller particle size. Thus, a particle of diameter 50 μm corresponding to curve 2 (the more so a particle of 100 μm) executes oscillations with an amplitude much smaller than the amplitude a of oscillations of the liquid.

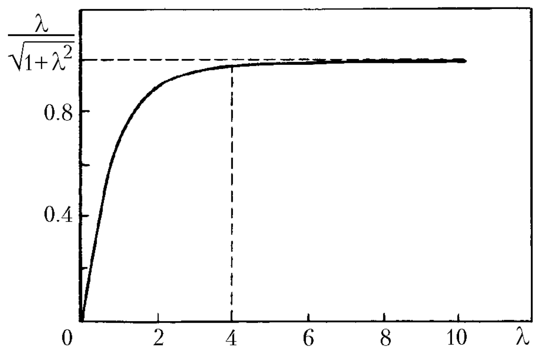
Figure 2 demonstrates the influence of the frequency of oscillations of the dispersion medium on the motion of a particle. A comparison of the curves calculated for different values of the frequency enables us to infer that the increase in the oscillation frequency enhances the lag of particles. The time of action on a particle is reduced in this case; it becomes shorter than the time of its relaxation.

Expression for the Amplitude of Oscillations of a Particle. It is of interest to find an expression for the amplitude of oscillations of a particle. An analysis of Eqs. (5) and (6) shows that the exponential term in them results from the noncoincidence of the velocities of the particle and the medium at the initial instant of time. It had been taken above that the particle is at rest at $t = 0$, whereas, according to (2), the liquid moves with a velocity $a\omega$. This asynchronism in motion disappears with time and the amplitude of particle oscillations becomes constant. Mathematically this means that the exponential term disappears from Eqs. (5) and (6); at $t \to \infty$, we have $\exp(k_1 t) \to 0$, since $k_1 < 0$.

Thus, particle oscillations in the steady-state regime are described by the following equations:

$$v(t) = -\frac{k_2}{k_1^2 + \omega^2}\left[k_1 \cos(\omega t) - \omega \sin(\omega t)\right], \tag{7}$$

$$y(t) = -\frac{k_2}{\omega\,(k_1^2 + \omega^2)}\left[k_1 \sin(\omega t) + \omega \cos(\omega t)\right]. \tag{8}$$



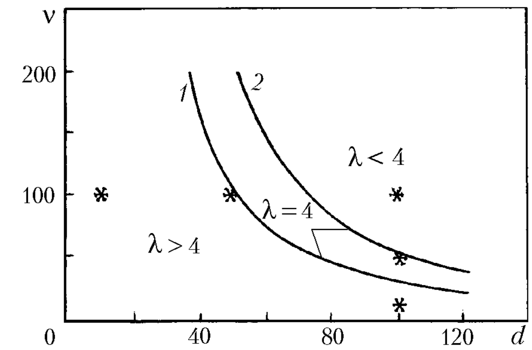


Fig. 3. Factor of proportionality $\lambda \sqrt{1+\lambda^2}$ between the amplitudes of oscillations of a particle and the medium as a function of the dimensionless parameter λ.

Fig. 4. Curves λ = 4 in the particle diameter–oscillation frequency coordinates for the quartz–water (1) (ρ = 2600 kg/m^3) and carbon–water (2) (ρ = 1350 kg/m^3) systems. The points correspond to the cases shown in Figs. 1 I and 2

The substitution of the solution of the equation $v(t) = 0$ that has been obtained by setting (7) equal to zero into (8) yields the amplitude values of the coordinate of the particle

$$A = \frac{k_2}{\omega\sqrt{k_1^2 + \omega^2}} = \frac{3\pi\eta d a}{m\omega\sqrt{1 + \left(\frac{3\pi\eta d}{m\omega}\right)^2}} = \frac{\lambda a}{\sqrt{1+\lambda^2}}. \tag{9}$$

The dimensionless parameter $\lambda = \frac{3\pi\eta d}{m\omega} = \frac{9\eta}{\pi d^2 \rho \nu}$ characterizes the capacity of the particle to react to the oscillations of the medium.

The factor of proportionality between the amplitudes of liquid and particle oscillations in Eq. (9) is equal to $\lambda/\sqrt{1+\lambda^2}$. Its dependence on the parameter λ is presented in Fig. 3, from which it is clear that the coincidence of the liquid and particle amplitudes is observed approximately for $\lambda > 4$. For $\lambda < 4$, the amplitude of particle oscillations becomes appreciably smaller than the amplitude of liquid oscillations; this difference is the more significant, the lower the λ. Figure 4 shows (in the coordinates ν–*d*) the curves obtained for λ = 4 for different disperse systems and subdividing of the coordinate plane into two regions. In the region below the curve, the particle oscillates with the same amplitude as the dispersion medium ($\lambda > 4$). In the region above the curve ($\lambda < 4$), we no longer observe such a coincidence and the lag of the particle behind the medium manifests itself. Figure 4 also plots the points corresponding to different cases given in Figs. 1 and 2. Each of these cases is characterized by its own diameter of particles and the frequency of oscillations of the quartz–water disperse system. It is seen that the difference between the particle oscillations and the liquid oscillations is the most pronounced only for the point with coordinates *d* = 100 μm and ν = 100 Hz, which is consistent with the results of Figs. 1 and 2.

It is noteworthy that the dependence of the factor of proportionality between the amplitudes of oscillations of the medium and the particle on the parameter λ is universal in character, i.e., is related to neither the properties of a specific disperse system nor the characteristics of vibration action. All these quantities are directly involved in the parameter λ; the relationship between λ and the amplitude ratio *A/a* remains constant.

Discussion of the Results. The importance of this reasoning lies in the fact that the dimensionless number λ is in essence the criterion from which we can judge the possibility of disintegration of the structures and aggregates in the dispersion. The parameter λ enables us to formulate two conditions on satisfying which we can draw a conclusion on the possibility of disintegration of structured formations: 1) the particles composing the disperse system must be characterized by different values of λ; the wider the range of variation of λ, the faster and more efficient the disintegration of the structure; 2) the dispersion must contain a certain number of particles with $\lambda < 4$.

The physical meaning of the first condition is that particles with a varying inertia toward external action, i.e., with a varying mass, must be contained in the system. If the particles composing the solid phase are homogeneous in density, the first condition means that it is only the polydisperse systems with a wide range of variation of particle diameters that are capable of disintegrating under the action of vibration. Variously sized particles tend to oscillate with different amplitudes, which generates a shift of the particles forming the structure relative to each other, causing the structure to disintegrate. What the second condition means is that an efficient structural disintegration requires that high-inertia particles whose oscillation amplitude is much smaller than the oscillation amplitude of the medium be part of the dispersion. It is precisely such particles that mainly destroy the structure, even such a structure that is formed by fine monodisperse particles (i.e., by particles of low and equal inertia). From Fig. 4 it is clear that the presence of particles with a size no smaller than 60 μm is necessary for the quartz–water system with an oscillation frequency of 100 Hz. If this system does not contain such large particles and consists of finer ones, even if of different size, disintegration of its structure under the action of vibration with a frequency of 100 Hz is impossible; it occurs at higher frequencies. For the carbon–water system, structural disintegration is even more complicated, i.e., it can be realized due to the higher-frequency oscillations and requires that larger particles be present.

It should be added to what has been said above that the possibility of structural disintegration can be determined by one condition if each particle in the system is characterized by the factor of proportionality $\beta = \lambda\sqrt{1+\lambda^2}$ between the amplitudes of oscillations of the liquid and the particle $A = \beta a$ and not by the parameter λ. The structure is capable of disintegrating if the particles forming it are characterized by different values of β. The second condition becomes unnecessary in such a formulation, since the relative difference of the particle amplitudes is already characterized directly by the parameter β. Thus, for two particles with different values of β we have

$$\frac{\beta_1 - \beta_2}{\beta_1} = \frac{A_1/a - A_2/a}{A_1/a} = \frac{A_1 - A_2}{A_1}\,.$$

It should be borne in mind that the quantities A_1 and A_2 are the oscillation amplitudes of free, i.e., not mutually interacting, particles; the actual amplitudes can be smaller because of the attractions between the particles. What this means is that the criteria formulated only point to the possibility of disintegration of structured formations. Whether the structure will actually disintegrate depends on its strength determined by the forces of interparticle interaction. To overcome these forces it is required that the energy supplied in the course of vibration action be sufficient for breaking of coagulation contacts [2–4]. In other words, the criteria obtained are necessary but not sufficient conditions for disintegration of the structure of S/L disperse systems.

Conclusions. Thus, when external vibration action by means of the oscillations of a dispersion medium is applied to the S/L disperse system, we can observe a lag of the particle oscillations behind the medium's oscillations in both amplitude and phase. This lag is the more considerable, the larger the mass of a particle, the higher the oscillation frequency of the medium, and the lower its viscosity. It is precisely owing to the fact that such a lag manifests itself differently for different particles that the disintegration of a coagulation structure formed by these particles is made possible. If all the particles of the system executed oscillations exactly "in time" with each other, structural disintegration under the action of vibration would be absolutely impossible. The necessary condition of disintegration of the dispersion structure can be formulated more rigorously if each particle in the disperse system is characterized by the factor of proportionality β between the amplitude of its natural oscillations and oscillations of the medium. This factor is a function of the properties of the disperse system and the parameters of vibration action. The disintegration condition is that the particles forming the coagulation structure must be characterized by different values of β. The more considerable is such a difference, the faster and more efficient is the disintegration of the structure.

In the present work, consideration has been given to systems consisting of particles of a regular spherical shape. The capacity of structures and aggregates for disintegrating under vibration action certainly depends on the degree of anisometry of the particles. The particle shape largely determines the value and character of the hydrodynamic forces acting on them. When the degree of anisometry is high, we have not only the relative displacement of particles but also a more complex translational–rotational motion of them, which results in a faster disintegration of aggregates.

This work was carried out with support from the Russian Foundation for Basic Research (project No. 03-03-32237).

NOTATION

a, amplitude of oscillations of a dispersion medium, m; A, amplitude of oscillations of a particle, m; d, diameter of a particle, μm; $k_1 = -3\pi\eta d/m$; $k_2 = -a\omega k_1$; m, effective mass of a particle, kg; m_p, mass of a particle, kg; t, time, sec; v, linear speed of particle oscillations, m/sec; v^{liq}, linear speed of dispersion-medium oscillations, m/sec; y, displacement of a particle in oscillations, m; y^{liq}, displacement of the dispersion medium in oscillations, m; β, factor of proportionality between the amplitudes of the medium and a particle; η, viscosity of the dispersion medium, Pa·sec; λ, dimensionless parameter; ν, frequency of oscillations of the medium, Hz; ρ, density of a particle, kg/m^3; ρ_{liq}, density of a dispersion medium, kg/m^3; τ_i, relaxation time of inertial motion of a particle, sec; τ_{liq}, period of oscillations of the dispersion medium, sec; ω, angular frequency of oscillations of the medium, rad/sec ($\omega = 2\pi\nu$). Subscripts and superscripts: liq, liquid; i, inertial motion; p, particle.

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